

Basic

TECHNICAL MATHEMATICS

with **CALCULUS**

Allyn J. Washington Richard S. Evans

Eleventh
Edition

 Pearson

ELEVENTH EDITION

Basic Technical Mathematics with Calculus

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Scope of the Book

Basic Technical Mathematics with Calculus, Eleventh Edition, is intended primarily for students in technical and pre-engineering technical programs or other programs for which coverage of mathematics is required. Chapters 1 through 20 provide the necessary background for further study with an integrated treatment of algebra and trigonometry. Chapter 21 covers the basic topics of analytic geometry, and Chapter 22 gives an introduction to statistics. Chapters 23 through 31 cover fundamental concepts of calculus including limits, derivatives, integrals, series representation of functions, and differential equations. In the examples and exercises, numerous applications from the various fields of technology are included, primarily to indicate where and how mathematical techniques are used. However, it is not necessary that the student have a specific knowledge of the technical area from which any given problem is taken. Most students using this text will have a background that includes some algebra and geometry. However, the material is presented in adequate detail for those who may need more study in these areas. The material presented here is sufficient for two to three semesters. One of the principal reasons for the arrangement of topics in this text is to present material in an order that allows a student to take courses concurrently in allied technical areas, such as physics and electricity. These allied courses normally require a student to know certain mathematics topics by certain definite times; yet the traditional order of topics in mathematics courses makes it difficult to attain this coverage without loss of continuity. However, the material in this book can be rearranged to fit any appropriate sequence of topics. The approach used in this text is not unduly rigorous mathematically, although all appropriate terms and concepts are introduced as needed and given an intuitive or algebraic foundation. The aim is to help the student develop an understanding of mathematical methods without simply providing a collection of formulas. The text material is developed recognizing that it is essential for the student to have a sound background in algebra and trigonometry in order to understand and succeed in any subsequent work in mathematics.

New to This Edition

You may have noticed something new on the cover of this book. Another author! Yes, after 50 years as a “solo act,” Allyn Washington has a partner. New co-author Rich Evans is a veteran faculty member at Corning Community College (NY) and has brought a wealth of positive contributions to the book and accompanying MyMathLab course.

The new features of the eleventh edition include:

- **Refreshed design** – The book has been redesigned in full color to help students better use it and to help motivate students as they put in the hard work to learn the mathematics (because let’s face it—a more modern looking book has more appeal).
- **Graphing calculator** – We have replaced the older TI-84 screens with those from the new TI-84 Plus-C (the color version). And Benjamin Rushing [Northwestern State University] has added graphing calculator help for students, accessible online via short URLs in the margins. If you’d like to see the complete listing of entries for the online graphing calculator manual, use the URL goo.gl/eAUgW3.
- **Applications** – The text features a wealth of new applications in the examples and exercises (over 200 in all!). Here is a sampling of the contexts for these new applications:
 - Power of a wind turbine (Section 3.4)
 - Height of One World Trade Center (Section 4.4)
 - GPS satellite velocity (Section 8.4)
 - Google’s self-driving car laser distance (Section 9.6)
 - Phase angle for current/voltage lead and lag (Section 10.3)
 - Growth of computer processor transistor counts (Section 13.7)

CAUTION When you enter URLs for the Graphing Calculator Manual, take care to distinguish the following characters:

l = lowercase l

I = uppercase I

1 = one

O = uppercase O

0 = zero ■

- Bezier curve roof design (Section 15.3)
- Cardioid microphone polar pattern (Section 21.7)
- Social networks usage (Section 22.1)
- Video game system market share (Section 22.1)
- Bluetooth headphone maximum revenue (Section 24.7)
- Saddledome roof slopes (Section 29.3)
- Weight loss differential equation (Section 31.6)
- **Exercises** – There are over 1000 new and updated exercises in the new edition. In creating new exercises, the authors analyzed aggregated student usage and performance data from MyMathLab for the previous edition of this text. The results of this analysis helped improve the quality and quantity of exercises that matter the most to instructors and students. There are a total of 14,000 exercises and 1400 examples in the eleventh edition.
- **Chapter Endmatter** – The exercises formerly called “Quick Chapter Review” are now labeled “Concept Check Exercises” (to better communicate their function within the chapter endmatter).
- **MyMathLab** – Features of the MyMathLab course for the new edition include:
 - Hundreds of new assignable algorithmic exercises help you address the homework needs of students. Additionally, all exercises are in the new HTML5 player, so they are accessible via mobile devices.
 - 223 new instructional videos (to augment the existing 203 videos) provide help for students as they do homework. These videos were created by Sue Glascoe (Mesa Community College) and Benjamin Rushing (Northwestern State University).
 - A new Graphing Calculator Manual, created specifically for this text, features instructions for the TI-84 and TI-89 family of calculators.
 - New PowerPoint® files feature animations that are designed to help you better teach key concepts.
 - Study skills modules help students with the life skills (e.g., time management) that can make the difference between passing and failing.

Content updates for the eleventh edition were informed by the extensive reviews of the text completed for this revision. These include:

- Unit analysis, including operations with units and unit conversions, has been moved from Appendix B to Section 1.4. Appendix B has been streamlined, but still contains the essential reference materials on units.
- In Section 1.3, more specific instructions have been provided for rounding combined operations with approximate numbers.
- Engineering notation has been added to Section 1.5.
- Finding the domain and range of a function *graphically* has been added to Section 3.4.
- The terms *input*, *output*, *piecewise defined functions*, and *practical domain and range* have been added to Chapter 3.
- In response to reviewer feedback, the beginning of Chapter 5 has been reorganized so that systems of equations has a strong introduction in Section 5.2. The prerequisite material needed for systems of equations (linear equations and graphs of linear functions) has been consolidated into Section 5.1. An example involving linear regression has also been added to Section 5.1.
- Solving systems using reduced row echelon form (*rref*) on a calculator has been added to Chapter 5.
- Several reviewers made the excellent suggestion to strengthen the focus on factoring in Chapter 6 by taking the contents of 6.1 (Special Products) and spreading it throughout the chapter. This change has been implemented. The terminology *greatest common factor* (GCF) has also been added to this chapter.

- In Chapter 7, the *square root property* is explicitly stated and illustrated.
- In Chapter 8, the unit circle definition of the trigonometric functions has been added.
- In Chapter 9, more emphasis had been given to solving equilibrium problems, including those that have more than one unknown.
- In Chapter 10, an example was added to show how the *phase angle* can be interpreted, and how it is different from the *phase shift*.
- In Chapter 16, the terminology *row echelon form* is used. Also, solving a system using *rref* is again illustrated. The material on using properties to evaluate determinants was deleted.
- The terminology *binomial coefficients* was added to Chapter 19.
- Chapter 22 (Introduction to Statistics) has undergone significant changes.
 - Section 22.1 now discusses common graphs used for both qualitative data (bar graphs and pie charts) and quantitative data (histograms, stem-and-leaf plots, and time series plots).
 - In Section 22.2, what was previously called the *arithmetic mean* is now referred to as simply the *mean*.
 - The *empirical rule* had been added to Section 22.4.
 - The sampling distribution of \bar{x} has been formalized including the statement of the *central limit theorem*.
 - A discussion of interpolation and extrapolation has been added in the context of regression, as well as information on how to interpret the values of r and r^2 .
 - The emphasis of Section 22.7 on nonlinear regression has been changed. Information on how to choose an appropriate type of model depending on the shape of the data has been added. However, a calculator is now used to obtain the actual regression equation.
- In Chapter 23, the terminology *direct substitution* has been introduced in the context of limits.
- Throughout the calculus chapters, many of the differentiation and integration rules have been given names so they can be easily referred to. These include, the *constant rule*, *power rule*, *constant multiple rule*, *product rule*, *quotient rule*, *general power rule*, *power rule for integration*, etc.
- In Chapter 30, the proof of the Fourier coefficients has been moved online.

Continuing Features

PAGE LAYOUT

Special attention has been given to the page layout. We specifically tried to avoid breaking examples or important discussions across pages. Also, all figures are shown immediately adjacent to the material in which they are discussed. Finally, we tried to avoid referring to equations or formulas by number when the referent is not on the same page spread.

CHAPTER INTRODUCTIONS

Each chapter introduction illustrates specific examples of how the development of technology has been related to the development of mathematics. In these introductions, it is shown that these past discoveries in technology led to some of the methods in mathematics, whereas in other cases mathematical topics already known were later very useful in bringing about advances in technology. Also, each chapter introduction contains a photo that refers to an example that is presented within that chapter.

WORKED-OUT EXAMPLES

EXAMPLE 3 Symbol in capital and in lowercase—forces on a beam

In the study of the forces on a certain beam, the equation $W = \frac{L(wL + 2P)}{8}$ is used.

Solve for P .

$$8W = \frac{8L(wL + 2P)}{8} \quad \text{multiply both sides by 8}$$

$$8W = L(wL + 2P) \quad \text{simplify right side}$$

$$8W = wL^2 + 2LP \quad \text{remove parentheses}$$

$$8W - wL^2 = 2LP \quad \text{subtract } wL^2 \text{ from both sides}$$

$$P = \frac{8W - wL^2}{2L} \quad \text{divide both sides by } 2L \text{ and switch sides} \quad \blacksquare$$

- **“HELP TEXT”** Throughout the book, special explanatory comments in blue type have been used in the examples to emphasize and clarify certain important points. Arrows are often used to indicate clearly the part of the example to which reference is made.
- **EXAMPLE DESCRIPTIONS** A brief descriptive title is given for each example. This gives an easy reference for the example, particularly when reviewing the contents of the section.

- **APPLICATION PROBLEMS** There are over 350 applied examples throughout the text that show complete solutions of application problems. Many relate to modern technology such as computer design, electronics, solar energy, lasers, fiber optics, the environment, and space technology. Others examples and exercises relate to technologies such as aeronautics, architecture, automotive, business, chemical, civil, construction, energy, environmental, fire science, machine, medical, meteorology, navigation, police, refrigeration, seismology, and wastewater. The Index of Applications at the end of the book shows the breadth of applications in the text.

KEY FORMULAS AND PROCEDURES

Throughout the book, important formulas are set off and displayed so that they can be easily referenced for use. Similarly, summaries of techniques and procedures consistently appear in color-shaded boxes.

“CAUTION” AND “NOTE” INDICATORS

CAUTION This heading is used to identify errors students commonly make or places where they frequently have difficulty. ■

The **NOTE** label in the side margin, along with accompanying blue brackets in the main body of the text, points out material that is of particular importance in developing or understanding the topic under discussion. [Both of these features have been clarified in the eleventh edition by adding a small design element to show where the **CAUTION** or **NOTE** feature *ends*.]

NOTE ▶

CHAPTER AND SECTION CONTENTS


A listing of learning outcomes for each chapter is given on the introductory page of the chapter. Also, a listing of the key topics of each section is given below the section number and title on the first page of the section. This gives the student and instructor a quick preview of the chapter and section contents.

PRACTICE EXERCISES

Most sections include some practice exercises in the margin. They are included so that a student is more actively involved in the learning process and can check his or her understanding of the material. They can also be used for classroom exercises. The answers to these exercises are given at the end of the exercises set for the section. There are over 450 of these exercises.

FEATURES OF EXERCISES

- **EXERCISES DIRECTLY REFERENCED TO TEXT EXAMPLES** The first few exercises in most of the text sections are referenced directly to a specific example of the section. These exercises are worded so that it is necessary for the student to refer to the example in order to complete the required solution. In this way, the student should be able to better review and understand the text material before attempting to solve the exercises that follow.

- **WRITING EXERCISES** There are over 270 writing exercises through the book (at least eight in each chapter) that require at least a sentence or two of explanation as part of the answer. These are noted by a pencil icon  next to the exercise number.
- **APPLICATION PROBLEMS** There are about 3000 application exercises in the text that represent the breadth of applications that students will encounter in their chosen professions. The Index of Applications at the end of the book shows the breadth of applications in the text.

CHAPTER ENDMATTER

- **KEY FORMULAS AND EQUATIONS** Here all important formulas and equations are listed together with their corresponding equation numbers for easy reference.
- **CHAPTER REVIEW EXERCISES** These exercises consist of (a) Concept Check Exercises (a set of true/false exercises) and (b) Practice and Applications.
- **CHAPTER TEST** These are designed to mirror what students might see on the actual chapter test. Complete step-by-step solutions to all practice test problems are given in the back of the book.

MARGIN NOTES

Throughout the text, some margin notes point out relevant historical events in mathematics and technology. Other margin notes are used to make specific comments related to the text material. Also, where appropriate, equations from earlier material are shown for reference in the margin.

ANSWERS TO EXERCISES

The answers to odd-numbered exercises are given near the end of the book. The Student's Solution Manual contains solutions to every other odd-numbered exercise and the Instructor's Solution Manual contains solutions to all section exercises.

FLEXIBILITY OF COVERAGE

The order of coverage can be changed in many places and certain sections may be omitted without loss of continuity of coverage. Users of earlier editions have indicated successful use of numerous variations in coverage. Any changes will depend on the type of course and completeness required.

Technology and Supplements

MYMATHLAB® ONLINE COURSE (ACCESS CODE REQUIRED)

Built around Pearson's best-selling content, MyMathLab is an online homework, tutorial, and assessment program designed to work with this text to engage students and improve results. MyMathLab can be successfully implemented in any classroom environment—lab-based, hybrid, fully online, or traditional. **By addressing instructor and student needs, MyMathLab improves student learning.**

MOTIVATION

Students are motivated to succeed when they're engaged in the learning experience and understand the relevance and power of mathematics. MyMathLab's online homework offers students immediate feedback and tutorial assistance that motivates them to do more, which means they retain more knowledge and improve their test scores.

- Exercises with immediate feedback**—over 7850 assignable exercises—are based on the textbook exercises, and regenerate algorithmically to give students unlimited opportunity for practice and mastery. MyMathLab provides helpful feedback when students enter incorrect answers and includes optional learning aids including Help Me Solve This, View an Example, videos, and the eText.

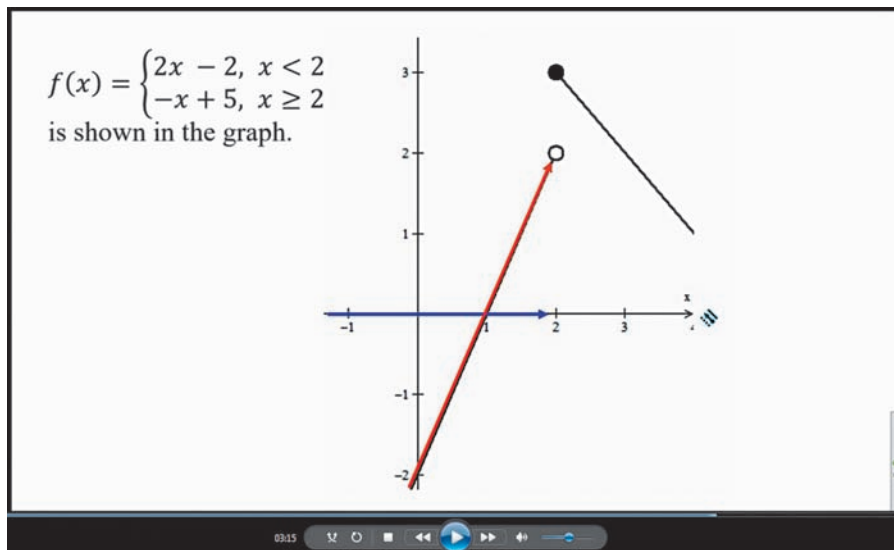
The screenshot shows a MyMathLab homework page titled "Homework: Quadratics". At the top right is a "Save" button. Below the title, it shows "Score: 0 of 1 pt" and "9 of 52 (0 complete)". The "HW Score" is "0% of 52 pts". The question ID is "8.3.9". The problem text says: "In the following exercise, find the coordinates of the vertex for the parabola defined by the given quadratic function." The function is $f(x) = 3(x - 6)^2 + 1$. The question asks: "The vertex is . (Type an ordered pair.)" A dropdown menu is open on the right with options: "Help Me Solve This", "View an Example", "Textbook", "Ask My Instructor", and "Print". At the bottom, there is a toolbar with mathematical symbols, a "Check Answer" button, and a "Clear All" button.

- Learning Catalytics™** is a student response tool that uses students' smartphones, tablets, or laptops to engage them in more interactive tasks and thinking. Learning Catalytics fosters student engagement and peer-to-peer learning with real-time analytics.

The screenshot shows the Learning Catalytics interface. The top header says "learning catalytics™" and "Alicia Frankel | Boston College | Log out". The navigation path is "My Courses > Washington Basic Tech Math 10e > Washington". The current session is "99237330 | 1 student". There are buttons for "Stop session", "Edit", and "Ask a new question on the fly". The task is labeled "sketch" and says "Graph the function." The function is $h(x) = (x - 1)^2$. A coordinate plane is shown with x and y axes ranging from -6 to 6. On the right, a smaller window shows the same task with a blue parabola graphed on the coordinate plane, opening upwards with its vertex at (1, 0).

LEARNING TOOLS FOR STUDENTS

- **Instructional videos** - The nearly 440 videos in the 11th edition MyMathLab course provide help for students outside of the classroom. These videos are also available as learning aids within the homework exercises, for students to refer to at point-of-use.



- **The complete eText** is available to students through their MyMathLab courses for the lifetime of the edition, giving students unlimited access to the eText within any course using that edition of the textbook. The eText includes links to videos.
- A new online **Graphing Calculator Manual**, created specifically for this text by Benjamin Rushing (Northwestern State University), features instructions for the TI-84 and TI-89 family of calculators.
- **Skills for Success Modules** help foster strong study skills in collegiate courses and prepare students for future professions. Topics include “Time Management” and “Stress Management”.
- **Accessibility** and achievement go hand in hand. MyMathLab is compatible with the JAWS screen reader, and enables multiple-choice and free-response problem types to be read and interacted with via keyboard controls and math notation input. MyMathLab also works with screen enlargers, including ZoomText, MAGic, and SuperNova. And, all MyMathLab videos have closed-captioning. More information is available at <http://mymathlab.com/accessibility>.

SUPPORT FOR INSTRUCTORS

- New **PowerPoint®** files feature animations that are designed to help you better teach key concepts.

When can we use the Cosine Law?

$$a^2 = b^2 + c^2 - 2bc(\cos A)$$

Case 3: Two Sides & the Angle between them (SAS)

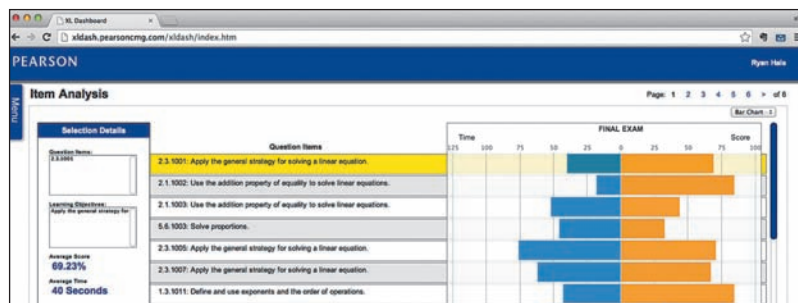
1. solve for the side opposite the known angle using the Cosine Law
2. solve for one of the two unknown angles using the Sine Law
3. solve for the 3rd unknown angle using the 3 angles add to 180°

So if we knew Angle A and sides b and c our calculations would be:

$$a^2 = b^2 + c^2 - 2bc(\cos A)$$

$$\frac{a}{\sin A} = \frac{b}{\sin B} \Rightarrow \sin B = \frac{b(\sin A)}{a}$$

$$C = 180^\circ - A - B$$



- A **comprehensive gradebook** with enhanced reporting functionality allows you to efficiently manage your course.
- **The Reporting Dashboard** provides insight to view, analyze, and report learning outcomes. Student performance data is presented at the class, section, and program levels in an accessible, visual manner so you'll have the information you need to keep your students on track.

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ISBN-10: 0134434633 | ISBN-13: 9780134434636

The Student's Solutions Manual by Matthew Hudelson (Washington State University) includes detailed solutions for every other odd-numbered section exercise. The manual is available in print and is downloadable from within MyMathLab.

INSTRUCTOR'S SOLUTIONS MANUAL (DOWNLOADABLE)

ISBN-10: 0134435893 | ISBN-13: 9780134435893

The Instructor's Solution Manual by Matthew Hudelson (Washington State University) contains detailed solutions to every section exercise, including review exercises. The manual is available to qualified instructors for download in the Pearson Instructor Resource www.pearsonhighered.com/irc or within MyMathLab.

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TestGen enables instructors to build, edit, print, and administer tests using a bank of questions developed to cover all objectives in the text. TestGen is algorithmically based, allowing you to create multiple but equivalent versions of the same question or test. Instructors can also modify test bank questions or add new questions. The TestGen software and accompanying test bank are available to qualified instructors for download in the Pearson Web Catalog www.pearsonhighered.com or within MyMathLab.

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Finally, we wish to sincerely thank again each of the over 375 reviewers of the eleven editions of this text. Their comments have helped further the education of more than two million students during since this text was first published in 1964.

Allyn Washington

Richard Evans

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Basic Algebraic Operations

Interest in things such as the land on which they lived, the structures they built, and the motion of the planets led people in early civilizations to keep records and to create methods of counting and measuring.

In turn, some of the early ideas of arithmetic, geometry, and trigonometry were developed. From such beginnings, mathematics has played a key role in the great advances in science and technology.

Often, mathematical methods were developed from scientific studies made in particular areas, such as astronomy and physics. Many people were interested in the math itself and added to what was then known. Although this additional mathematical knowledge may not have been related to applications at the time it was developed, it often later became useful in applied areas.

In the chapter introductions that follow, examples of the interaction of technology and mathematics are given. From these examples and the text material, it is hoped you will better understand the important role that math has had and still has in technology. In this text, there are applications from technologies including (but not limited to) aeronautical, business, communications, electricity, electronics, engineering, environmental, heat and air conditioning, mechanical, medical, meteorology, petroleum, product design, solar, and space.

We begin by reviewing the concepts that deal with numbers and symbols. This will enable us to develop topics in algebra, an understanding of which is essential for progress in other areas such as geometry, trigonometry, and calculus.

1

LEARNING OUTCOMES

After completion of this chapter, the student should be able to:

- Identify real, imaginary, rational, and irrational numbers
- Perform mathematical operations on integers, decimals, fractions, and radicals
- Use the fundamental laws of algebra in numeric and algebraic expressions
- Employ mathematical order of operations
- Understand technical measurement, approximation, the use of significant digits, and rounding
- Use scientific and engineering notations
- Convert units of measurement
- Rearrange and solve basic algebraic equations
- Interpret word problems using algebraic symbols



◀ **From the Great Pyramid of Giza, built in Egypt 4500 years ago, to the modern technology of today, mathematics has played a key role in the advancement of civilization. Along the way, important discoveries have been made in areas such as architecture, navigation, transportation, electronics, communication, and astronomy. Mathematics will continue to pave the way for new discoveries.**

1.1 Numbers

Real Number System • Number Line • Absolute Value • Signs of Inequality • Reciprocal • Denominate Numbers • Literal Numbers

■ Irrational numbers were discussed by the Greek mathematician Pythagoras in about 540 B.C.E.

■ For reference, $\pi = 3.14159265 \dots$

■ A notation that is often used for repeating decimals is to place a bar over the digits that repeat. Using this notation we can write $\frac{1121}{1665} = 0.\overline{6732}$ and $\frac{2}{3} = 0.\overline{6}$.

In technology and science, as well as in everyday life, we use the very familiar **counting numbers**, or **natural numbers** 1, 2, 3, and so on. The **whole numbers** include 0 as well as all the natural numbers. Because it is necessary and useful to use negative numbers as well as positive numbers in mathematics and its applications, the natural numbers are called the **positive integers**, and the numbers $-1, -2, -3,$ and so on are the **negative integers**.

Therefore, the **integers** include the positive integers, the negative integers, and **zero**, which is neither positive nor negative. This means that the integers are the numbers $\dots, -3, -2, -1, 0, 1, 2, 3, \dots$ and so on.

A **rational number** is a number that can be expressed as the division of one integer a by another nonzero integer b , and can be represented by the fraction a/b . Here a is the **numerator** and b is the **denominator**. Here we have used algebra by letting letters represent numbers.

Another type of number, an **irrational number**, cannot be written in the form of a fraction that is the division of one integer by another integer. The following example illustrates integers, rational numbers, and irrational numbers.

EXAMPLE 1 Identifying rational numbers and irrational numbers

The numbers 5 and -19 are integers. They are also rational numbers because they can be written as $\frac{5}{1}$ and $\frac{-19}{1}$, respectively. Normally, we do not write the 1's in the denominators.

The numbers $\frac{5}{8}$ and $\frac{-11}{3}$ are rational numbers because the numerator and the denominator of each are integers.

The numbers $\sqrt{2}$ and π are irrational numbers. It is not possible to find two integers, one divided by the other, to represent either of these numbers. In decimal form, irrational numbers are nonterminating, nonrepeating decimals. It can be shown that square roots (and other roots) that cannot be expressed exactly in decimal form are irrational. Also, $\frac{22}{7}$ is sometimes used as an *approximation* for π , but it is not equal *exactly* to π . We must remember that $\frac{22}{7}$ is rational and π is irrational.

The decimal number 1.5 is rational since it can be written as $\frac{3}{2}$. Any such *terminating decimal* is rational. The number $0.6666 \dots$, where the 6's continue on indefinitely, is rational because we may write it as $\frac{2}{3}$. In fact, any *repeating decimal* (in decimal form, a specific sequence of digits is repeated indefinitely) is rational. The decimal number $0.6732732732 \dots$ is a repeating decimal where the sequence of digits 732 is repeated indefinitely ($0.6732732732 \dots = \frac{1121}{1665}$). ■

The rational numbers together with the irrational numbers, including all such numbers that are positive, negative, or zero, make up the **real number system** (see Fig. 1.1). There are times we will encounter an **imaginary number**, the name given to the square root of a

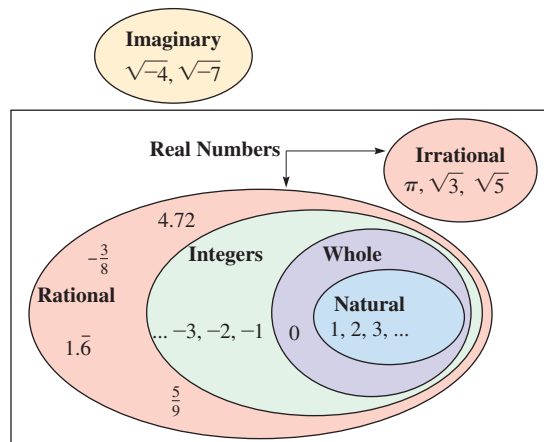


Fig. 1.1

negative number. Imaginary numbers are not real numbers and will be discussed in Chapter 12. However, unless specifically noted, we will use real numbers. Until Chapter 12, it will be necessary to only *recognize* imaginary numbers when they occur.

Also in Chapter 12, we will consider **complex numbers**, which include both the real numbers and imaginary numbers. See Exercise 39 of this section.

■ Real numbers and imaginary numbers are both included in the *complex number system*. See Exercise 39.

EXAMPLE 2 Identifying real numbers and imaginary numbers

- (a) The number 7 is an integer. It is also rational because $7 = \frac{7}{1}$, and it is a real number since the real numbers include all the rational numbers.
- (b) The number 3π is irrational, and it is real because the real numbers include all the irrational numbers.
- (c) The numbers $\sqrt{-10}$ and $-\sqrt{-7}$ are imaginary numbers.
- (d) The number $\frac{-3}{7}$ is rational and real. The number $-\sqrt{7}$ is irrational and real.
- (e) The number $\frac{\pi}{6}$ is irrational and real. The number $\frac{\sqrt{-3}}{2}$ is imaginary. ■

A **fraction** may contain any number or symbol representing a number in its numerator or in its denominator. The fraction indicates the division of the numerator by the denominator, as we previously indicated in writing rational numbers. Therefore, a fraction may be a number that is rational, irrational, or imaginary.

■ Fractions were used by early Egyptians and Babylonians. They were used for calculations that involved parts of measurements, property, and possessions.

EXAMPLE 3 Fractions

- (a) The numbers $\frac{2}{7}$ and $\frac{-3}{2}$ are fractions, and they are rational.
- (b) The numbers $\frac{\sqrt{2}}{9}$ and $\frac{6}{\pi}$ are fractions, but they are not rational numbers. It is not possible to express either as one integer divided by another integer.
- (c) The number $\frac{\sqrt{-5}}{6}$ is a fraction, and it is an imaginary number. ■

THE NUMBER LINE

Real numbers may be represented by points on a line. We draw a horizontal line and designate some point on it by O , which we call the **origin** (see Fig. 1.2). The integer *zero* is located at this point. Equal intervals are marked to the right of the origin, and the positive integers are placed at these positions. The other positive rational numbers are located between the integers. The points that cannot be defined as rational numbers represent irrational numbers. We cannot tell whether a given point represents a rational number or an irrational number unless it is specifically marked to indicate its value.

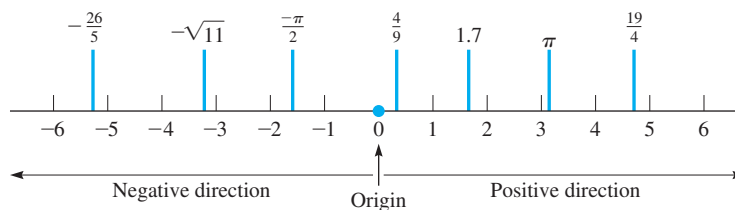


Fig. 1.2

The negative numbers are located on the number line by starting at the origin and marking off equal intervals *to the left*, which is the **negative direction**. As shown in Fig. 1.2, the positive numbers are to the right of the origin and the negative numbers are to the left of the origin. Representing numbers in this way is especially useful for graphical methods.

We next define another important concept of a number. The **absolute value** of a positive number is the number itself, and the absolute value of a negative number is the corresponding positive number. On the number line, we may interpret the absolute value of a number as the distance (which is always positive) between the origin and the number. Absolute value is denoted by writing the number between vertical lines, as shown in the following example.

EXAMPLE 4 Absolute value

The absolute value of 6 is 6, and the absolute value of -7 is 7. We write these as $|6| = 6$ and $|-7| = 7$. See Fig. 1.3.

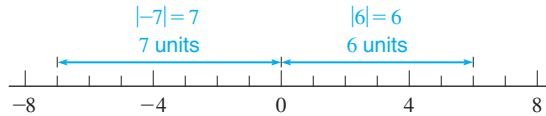


Fig. 1.3

Practice Exercises

1. $|-4.2| = ?$ 2. $-\left|-\frac{3}{4}\right| = ?$

■ The symbols $=$, $<$, and $>$ were introduced by English mathematicians in the late 1500s.

Other examples are $\left|\frac{7}{5}\right| = \frac{7}{5}$, $|\sqrt{2}| = \sqrt{2}$, $|0| = 0$, $|\pi| = \pi$, $|-5.29| = 5.29$, and $|-9| = 9$ since $|-9| = 9$. ■

On the number line, if a first number is to the right of a second number, then the first number is said to be **greater than** the second. If the first number is to the left of the second, it is **less than** the second number. The symbol $>$ designates “is greater than,” and the symbol $<$ designates “is less than.” These are called **signs of inequality**. See Fig. 1.4.

EXAMPLE 5 Signs of inequality

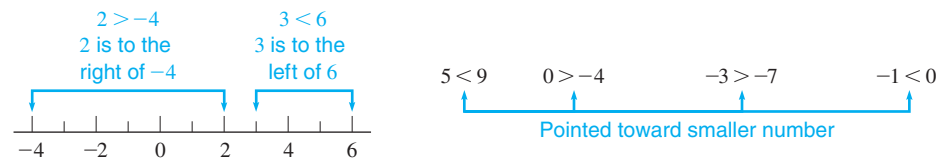


Fig. 1.4

Practice Exercises

Place the correct sign of inequality ($<$ or $>$) between the given numbers.

3. -5 4. 4 4. 0 -3

Every number, except zero, has a **reciprocal**. The reciprocal of a number is 1 divided by the number.

EXAMPLE 6 Reciprocal

The reciprocal of 7 is $\frac{1}{7}$. The reciprocal of $\frac{2}{3}$ is

$$\frac{1}{\frac{2}{3}} = 1 \times \frac{3}{2} = \frac{3}{2} \quad \text{invert denominator and multiply (from arithmetic)}$$

The reciprocal of 0.5 is $\frac{1}{0.5} = 2$. The reciprocal of $-\pi$ is $-\frac{1}{\pi}$. Note that the negative sign is retained in the reciprocal of a negative number.

We showed the multiplication of 1 and $\frac{3}{2}$ as $1 \times \frac{3}{2}$. We could also show it as $1 \cdot \frac{3}{2}$ or $1\left(\frac{3}{2}\right)$. We will often find the form with parentheses is preferable. ■

Practice Exercise

5. Find the reciprocals of
 (a) -4 (b) $\frac{3}{8}$

In applications, numbers that represent a measurement and are written with units of measurement are called **denominate numbers**. The next example illustrates the use of units and the symbols that represent them.

■ For reference, see Appendix B for units of measurement and the symbols used for them.

EXAMPLE 7 Denominate numbers

- (a) To show that a certain TV weighs 62 pounds, we write the weight as 62 lb.
- (b) To show that a giant redwood tree is 330 feet high, we write the height as 300 ft.
- (c) To show that the speed of a rocket is 1500 meters per second, we write the speed as 1500 m/s. (Note the use of s for second. We use s rather than sec.)
- (d) To show that the area of a computer chip is 0.75 square inch, we write the area as 0.75 in.². (We will not use sq in.)
- (e) To show that the volume of water in a glass tube is 25 cubic centimeters, we write the volume as 25 cm³. (We will not use cu cm nor cc.) ■

It is usually more convenient to state definitions and operations on numbers in a general form. *To do this, we represent the numbers by letters, called **literal numbers**.* For example, if we want to say “If a first number is to the right of a second number on the number line, then the first number is greater than the second number,” we can write “If a is to the right of b on the number line, then $a > b$.” Another example of using a literal number is “The reciprocal of n is $1/n$.”

Certain literal numbers may take on any allowable value, whereas other literal numbers represent the same value throughout the discussion. *Those literal numbers that may vary in a given problem are called **variables**, and those literal numbers that are held fixed are called **constants**.*

EXAMPLE 8 Variables and constants

- (a) The resistance of an electric resistor is R . The current I in the resistor equals the voltage V divided by R , written as $I = V/R$. For this resistor, I and V may take on various values, and R is fixed. This means I and V are variables and R is a constant. For a *different* resistor, the value of R may differ.
- (b) The fixed cost for a calculator manufacturer to operate a certain plant is b dollars per day, and it costs a dollars to produce each calculator. The total daily cost C to produce n calculators is

$$C = an + b$$

Here, C and n are variables, and a and b are constants, and the product of a and n is shown as an . For *another* plant, the values of a and b would probably differ.

If specific numerical values of a and b are known, say $a = \$7$ per calculator and $b = \$3000$, then $C = 7n + 3000$. Thus, constants may be numerical or literal. ■

EXERCISES 1.1

In Exercises 1–4, make the given changes in the indicated examples of this section, and then answer the given questions.

- In the first line of Example 1, change the 5 to -7 and the -19 to 12. What other changes must then be made in the first paragraph?
- In Example 4, change the 6 to -6 . What other changes must then be made in the first paragraph?
- In the left figure of Example 5, change the 2 to -6 . What other changes must then be made?
- In Example 6, change the $\frac{2}{3}$ to $\frac{3}{2}$. What other changes must then be made?

In Exercises 5–8, designate each of the given numbers as being an integer, rational, irrational, real, or imaginary. (More than one designation may be correct.)

- $3, \sqrt{-4}$
- $\frac{\sqrt{7}}{3}, -6$
- $-\frac{\pi}{6}, \frac{1}{8}$
- $-\sqrt{-6}, -2.33$

In Exercises 9 and 10, find the absolute value of each real number.

- $3, -3, \frac{\pi}{4}, \sqrt{-1}$
- $-0.857, \sqrt{2}, -\frac{19}{4}, \frac{\sqrt{-5}}{-2}$

In Exercises 11–18, insert the correct sign of inequality ($>$ or $<$) between the given numbers.

11. 6 8

12. 7 5

13. π 3.1416

14. -4 0

15. -4 $-|-3|$

16. $-\sqrt{2}$ -1.42

17. $-\frac{2}{3}$ $-\frac{3}{4}$

18. -0.6 0.2

In Exercises 19 and 20, find the reciprocal of each number.

19. 3, $-\frac{4}{\sqrt{3}}$, $\frac{y}{b}$

20. $-\frac{1}{3}$, 0.25, $2x$

In Exercises 21 and 22, locate (approximately) each number on a number line as in Fig. 1.2.

21. 2.5, $-\frac{12}{5}$, $\sqrt{3}$, $-\frac{3}{4}$

22. $-\frac{\sqrt{2}}{2}$, 2π , $\frac{123}{19}$, $-\frac{7}{3}$

In Exercises 23–46, solve the given problems. Refer to Appendix B for units of measurement and their symbols.

23. Is an absolute value always positive? Explain.

24. Is -2.17 rational? Explain.

25. What is the reciprocal of the reciprocal of any positive or negative number?

26. Is the repeating decimal $2.\overline{72}$ rational or irrational?

27. True or False: A nonterminating, nonrepeating decimal is an irrational number.

28. If $b > a$ and $a > 0$, is $|b - a| < |b| - |a|$?

29. List the following numbers in numerical order, starting with the smallest: -1 , 9 , π , $\sqrt{5}$, $|-8|$, $-|-3|$, -3.1 .

30. List the following numbers in numerical order, starting with the smallest: $\frac{1}{5}$, $-\sqrt{10}$, $-|-6|$, -4 , 0.25 , $|\pi|$.

31. If a and b are positive integers and $b > a$, what type of number is represented by the following?

(a) $b - a$

(b) $a - b$

(c) $\frac{b - a}{b + a}$

32. If a and b represent positive integers, what kind of number is represented by (a) $a + b$, (b) a/b , and (c) $a \times b$?

33. For any positive or negative integer: (a) Is its absolute value always an integer? (b) Is its reciprocal always a rational number?

34. For any positive or negative rational number: (a) Is its absolute value always a rational number? (b) Is its reciprocal always a rational number?

35. Describe the location of a number x on the number line when (a) $x > 0$ and (b) $x < -4$.

36. Describe the location of a number x on the number line when (a) $|x| < 1$ and (b) $|x| > 2$.

37. For a number $x > 1$, describe the location on the number line of the reciprocal of x .

38. For a number $x < 0$, describe the location on the number line of the number with a value of $|x|$.

39. A complex number is defined as $a + bj$, where a and b are real numbers and $j = \sqrt{-1}$. For what values of a and b is the complex number $a + bj$ a real number? (All real numbers and all imaginary numbers are also complex numbers.)

40. A sensitive gauge measures the total weight w of a container and the water that forms in it as vapor condenses. It is found that $w = c\sqrt{0.1t + 1}$, where c is the weight of the container and t is the time of condensation. Identify the variables and constants.

41. In an electric circuit, the reciprocal of the total capacitance of two capacitors in series is the sum of the reciprocals of the capacitances $\left(\frac{1}{C_T} = \frac{1}{C_1} + \frac{1}{C_2}\right)$. Find the total capacitance of two capacitances of 0.0040 F and 0.0010 F connected in series.

42. Alternating-current (ac) voltages change rapidly between positive and negative values. If a voltage of 100 V changes to -200 V, which is greater in absolute value?

43. The memory of a certain computer has a bits in each byte. Express the number N of bits in n kilobytes in an equation. (A *bit* is a single digit, and bits are grouped in *bytes* in order to represent special characters. Generally, there are 8 bits per byte. If necessary, see Appendix B for the meaning of *kilo*.)

44. The computer design of the base of a truss is x ft long. Later it is redesigned and shortened by y in. Give an equation for the length L , in inches, of the base in the second design.

45. In a laboratory report, a student wrote “ $-20^\circ\text{C} > -30^\circ\text{C}$.” Is this statement correct? Explain.

46. After 5 s, the pressure on a valve is less than 60 lb/in.² (pounds per square inch). Using t to represent time and p to represent pressure, this statement can be written “for $t > 5$ s, $p < 60$ lb/in.².” In this way, write the statement “when the current I in a circuit is less than 4 A, the resistance R is greater than 12Ω (ohms).”

Answers to Practice Exercises

1. 4.2 2. $-\frac{3}{4}$ 3. $<$ 4. $>$ 5. (a) $-\frac{1}{4}$ (b) $\frac{8}{3}$

1.2 Fundamental Operations of Algebra

Fundamental Laws of Algebra •
Operations on Positive and Negative
Numbers • Order of Operations •
Operations with Zero

If two numbers are added, it does not matter in which order they are added. (For example, $5 + 3 = 8$ and $3 + 5 = 8$, or $5 + 3 = 3 + 5$.) This statement, generalized and accepted as being correct for all possible combinations of numbers being added, is called the **commutative law** for addition. It states that *the sum of two numbers is the same*,

regardless of the order in which they are added. We make no attempt to prove this law in general, but accept that it is true.

In the same way, we have the **associative law** for addition, which states that *the sum of three or more numbers is the same, regardless of the way in which they are grouped for addition*. For example, $3 + (5 + 6) = (3 + 5) + 6$.

The laws just stated for addition are also true for multiplication. Therefore, *the product of two numbers is the same, regardless of the order in which they are multiplied, and the product of three or more numbers is the same, regardless of the way in which they are grouped for multiplication*. For example, $2 \times 5 = 5 \times 2$, and $5 \times (4 \times 2) = (5 \times 4) \times 2$.

Another very important law is the **distributive law**. It states that *the product of one number and the sum of two or more other numbers is equal to the sum of the products of the first number and each of the other numbers of the sum*. For example,

■ Note carefully the difference:
 associative law: $5 \times (4 \times 2)$
 distributive law: $5 \times (4 + 2)$

$$5(4 + 2) = 5 \times 4 + 5 \times 2$$

In this case, it can be seen that the total is 30 on each side.

In practice, we use these **fundamental laws of algebra** naturally without thinking about them, except perhaps for the distributive law.

Not all operations are commutative and associative. For example, division is not commutative, because the order of division of two numbers does matter. For instance, $\frac{6}{5} \neq \frac{5}{6}$ (\neq is read “does not equal”). (Also, see Exercise 54.)

Using literal numbers, the fundamental laws of algebra are as follows:

Commutative law of addition: $a + b = b + a$

Associative law of addition: $a + (b + c) = (a + b) + c$

Commutative law of multiplication: $ab = ba$

Associative law of multiplication: $a(bc) = (ab)c$

Distributive law: $a(b + c) = ab + ac$

■ Note the meaning of *identity*.

Each of these laws is an example of an *identity*, in that the expression to the left of the = sign equals the expression to the right for any value of each of a , b , and c .

OPERATIONS ON POSITIVE AND NEGATIVE NUMBERS

When using the basic operations (addition, subtraction, multiplication, division) on positive and negative numbers, we determine the result to be either positive or negative according to the following rules.

Addition of two numbers of the same sign *Add their absolute values and assign the sum their common sign.*

EXAMPLE 1 Adding numbers of the same sign

- (a) $2 + 6 = 8$ the sum of two positive numbers is positive
- (b) $-2 + (-6) = -(2 + 6) = -8$ the sum of two negative numbers is negative

■ From Section 1.1, we recall that a positive number is preceded by no sign. Therefore, in using these rules, we show the “sign” of a positive number by simply writing the number itself.

The negative number -6 is placed in parentheses because it is also preceded by a plus sign showing addition. It is not necessary to place the -2 in parentheses. ■

Addition of two numbers of different signs Subtract the number of smaller absolute value from the number of larger absolute value and assign to the result the sign of the number of larger absolute value.

EXAMPLE 2 Adding numbers of different signs

- (a) $2 + (-6) = -(6 - 2) = -4$ ← the negative 6 has the larger absolute value
 (b) $-6 + 2 = -(6 - 2) = -4$ ←
 (c) $6 + (-2) = 6 - 2 = 4$ ← the positive 6 has the larger absolute value
 (d) $-2 + 6 = 6 - 2 = 4$ ←
 ↑ the subtraction of absolute values

Subtraction of one number from another Change the sign of the number being subtracted and change the subtraction to addition. Perform the addition.

EXAMPLE 3 Subtracting positive and negative numbers

(a) $2 - 6 = 2 + (-6) = -(6 - 2) = -4$

Note that after changing the subtraction to addition, and changing the sign of 6 to make it -6 , we have precisely the same illustration as Example 2(a).

(b) $-2 - 6 = -2 + (-6) = -(2 + 6) = -8$

Note that after changing the subtraction to addition, and changing the sign of 6 to make it -6 , we have precisely the same illustration as Example 1(b).

(c) $-a - (-a) = -a + a = 0$

NOTE ↗

This shows that subtracting a number from itself results in zero, even if the number is negative. [Subtracting a negative number is equivalent to adding a positive number of the same absolute value.]

(d) $-2 - (-6) = -2 + 6 = 4$

(e) The change in temperature from -12°C to -26°C is
 $-26^\circ\text{C} - (-12^\circ\text{C}) = -26^\circ\text{C} + 12^\circ\text{C} = -14^\circ\text{C}$

Multiplication and division of two numbers The product (or quotient) of two numbers of the same sign is positive. The product (or quotient) of two numbers of different signs is negative.

EXAMPLE 4 Multiplying and dividing positive and negative numbers

- (a) $3(12) = 3 \times 12 = 36$ $\frac{12}{3} = 4$ result is positive if both numbers are positive
 (b) $-3(-12) = 3 \times 12 = 36$ $\frac{-12}{-3} = 4$ result is positive if both numbers are negative
 (c) $3(-12) = -(3 \times 12) = -36$ $\frac{-12}{3} = -\frac{12}{3} = -4$ result is negative if one number is positive and the other is negative
 (d) $-3(12) = -(3 \times 12) = -36$ $\frac{12}{-3} = -\frac{12}{3} = -4$

Practice Exercises

- Evaluate: 1. $-5 - (-8)$
 2. $-5(-8)$

ORDER OF OPERATIONS

Often, how we are to combine numbers is clear by grouping the numbers using symbols such as **parentheses**, (); the **bar**, $\frac{\quad}{\quad}$, between the numerator and denominator of a fraction; and **vertical lines** for absolute value. Otherwise, for an expression in which there are several operations, we use the following order of operations.

Order of Operations

1. Perform operations within grouping symbols (parentheses, brackets, or absolute value symbols).
2. Perform multiplications and divisions (from left to right).
3. Perform additions and subtractions (from left to right).

■ Note that $20 \div (2 + 3) = \frac{20}{2+3}$, whereas $20 \div 2 + 3 = \frac{20}{2} + 3$.

EXAMPLE 5 Order of operations

- (a) $20 \div (2 + 3)$ is evaluated by first adding $2 + 3$ and then dividing. The grouping of $2 + 3$ is clearly shown by the parentheses. Therefore, $20 \div (2 + 3) = 20 \div 5 = 4$.
- (b) $20 \div 2 + 3$ is evaluated by first dividing 20 by 2 and then adding. No specific grouping is shown, and therefore the division is done before the addition. This means $20 \div 2 + 3 = 10 + 3 = 13$.
- NOTE (c) $[16 - 2 \times 3]$ is evaluated by *first multiplying* 2 by 3 and then subtracting. We do not first subtract 2 from 16. Therefore, $16 - 2 \times 3 = 16 - 6 = 10$.
- (d) $16 \div 2 \times 4$ is evaluated by first dividing 16 by 2 and then multiplying. From left to right, the division occurs first. Therefore, $16 \div 2 \times 4 = 8 \times 4 = 32$.
- (e) $|3 - 5| - |-3 - 6|$ is evaluated by first performing the subtractions within the absolute value vertical bars, then evaluating the absolute values, and then subtracting. This means that $|3 - 5| - |-3 - 6| = |-2| - |-9| = 2 - 9 = -7$. ■

When evaluating expressions, it is generally more convenient to change the operations and numbers so that the result is found by the addition and subtraction of positive numbers. When this is done, we must remember that

$$a + (-b) = a - b \quad (1.1)$$

$$a - (-b) = a + b \quad (1.2)$$

Practice Exercises

Evaluate: 3. $12 - 6 \div 2$

4. $16 \div (2 \times 4)$

EXAMPLE 6 Evaluating numerical expressions

- (a) $7 + (-3) - 6 = 7 - 3 - 6 = 4 - 6 = -2$ using Eq. (1.1)
- (b) $\frac{18}{-6} + 5 - (-2)(3) = -3 + 5 - (-6) = 2 + 6 = 8$ using Eq. (1.2)
- (c) $\frac{|3 - 15|}{-2} - \frac{8}{4 - 6} = \frac{12}{-2} - \frac{8}{-2} = -6 - (-4) = -6 + 4 = -2$
- (d) $\frac{-12}{2 - 8} + \frac{5 - 1}{2(-1)} = \frac{-12}{-6} + \frac{4}{-2} = 2 + (-2) = 2 - 2 = 0$

In illustration (b), we see that the division and multiplication were done before the addition and subtraction. In (c) and (d), we see that the groupings were evaluated first. Then we did the divisions, and finally the subtraction and addition. ■

EXAMPLE 7 Evaluating—velocity after collision

A 3000-lb van going at 40 mi/h ran head-on into a 2000-lb car going at 20 mi/h. An insurance investigator determined the velocity of the vehicles immediately after the collision from the following calculation. See Fig. 1.5.

$$\begin{aligned} \frac{3000(40) + (2000)(-20)}{3000 + 2000} &= \frac{120,000 + (-40,000)}{3000 + 2000} = \frac{120,000 - 40,000}{5000} \\ &= \frac{80,000}{5000} = 16 \text{ mi/h} \end{aligned}$$

The numerator and the denominator must be evaluated before the division is performed. The multiplications in the numerator are performed first, followed by the addition in the denominator and the subtraction in the numerator. ■

Practice Exercises

Evaluate: 5. $2(-3) - \frac{4 - 8}{2}$

6. $\frac{|5 - 15|}{2} - \frac{-9}{3}$

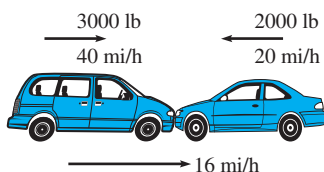


Fig. 1.5

OPERATIONS WITH ZERO

Because operations with zero tend to cause some difficulty, we will show them here.

If a is a real number, the operations of addition, subtraction, multiplication, and division with zero are as follows:

$$\begin{aligned}
 a + 0 &= a \\
 a - 0 &= a & 0 - a &= -a \\
 a \times 0 &= 0 \\
 0 \div a &= \frac{0}{a} = 0 \quad (\text{if } a \neq 0) & \neq & \text{means "is not equal to"}
 \end{aligned}$$

EXAMPLE 8 Operations with zero

(a) $5 + 0 = 5$ (b) $-6 - 0 = -6$ (c) $0 - 4 = -4$
 (d) $\frac{0}{6} = 0$ (e) $\frac{0}{-3} = 0$ (f) $\frac{5 \times 0}{7} = \frac{0}{7} = 0$ ■

Note that there is no result defined for division by zero. To understand the reason for this, consider the results for $\frac{6}{2}$ and $\frac{6}{0}$.

$$\frac{6}{2} = 3 \quad \text{since} \quad 2 \times 3 = 6$$

If $\frac{6}{0} = b$, then $0 \times b = 6$. This cannot be true because $0 \times b = 0$ for any value of b . Thus, **division by zero is undefined**.

(The special case of $\frac{0}{0}$ is termed *indeterminate*. If $\frac{0}{0} = b$, then $0 = 0 \times b$, which is true for any value of b . Therefore, no specific value of b can be determined.)

EXAMPLE 9 Division by zero is undefined

$\frac{2}{5} \div 0$ is undefined $\frac{8}{0}$ is undefined $\frac{7 \times 0}{0 \times 6}$ is indeterminate ■

[see above](#)

CAUTION The operations with zero will not cause any difficulty if we remember to *never divide by zero*. ■

Division by zero is the only undefined basic operation. All the other operations with zero may be performed as for any other number.

EXERCISES 1.2

In Exercises 1–4, make the given changes in the indicated examples of this section, and then solve the resulting problems.

In Exercises 5–38, evaluate each of the given expressions by performing the indicated operations.

1. In Example 5(c), change 3 to (-2) and then evaluate.
2. In Example 6(b), change 18 to -18 and then evaluate.
3. In Example 6(d), interchange the 2 and 8 in the first denominator and then evaluate.
4. In the rightmost illustration in Example 9, interchange the 6 and the 0 above the 6. Is any other change needed?

- | | | |
|--------------------|------------------------------|-----------------------------|
| 5. $5 + (-8)$ | 6. $-4 + (-7)$ | 7. $-3 + 9$ |
| 8. $18 - 21$ | 9. $-19 - (-16)$ | 10. $-8 - (-10)$ |
| 11. $7(-4)$ | 12. $-9(3)$ | 13. $-7(-5)$ |
| 14. $\frac{-9}{3}$ | 15. $\frac{-6(20 - 10)}{-3}$ | 16. $\frac{-28}{-7(5 - 6)}$ |

17. $-2(4)(-5)$ 18. $-3(-4)(-6)$ 19. $2(2 - 7) \div 10$
 20. $\frac{-64}{-2|4 - 8|}$ 21. $16 \div 2(-4)$ 22. $-20 \div 5(-4)$
 23. $-9 - |2 - 10|$ 24. $(7 - 7) \div (5 - 7)$ 25. $\frac{17 - 7}{7 - 7}$
 26. $\frac{(7 - 7)(2)}{(7 - 7)(-1)}$ 27. $8 - 3(-4)$ 28. $-20 + 8 \div 4$
 29. $-2(-6) + \left| \frac{8}{-2} \right|$ 30. $\frac{|-2|}{-2} - (-2)(-5)$
 31. $10(-8)(-3) \div (10 - 50)$ 32. $\frac{7 - |-5|}{-1(-2)}$
 33. $\frac{24}{3 + (-5)} - 4(-9) \div (-3)$ 34. $\frac{-18}{3} - \frac{4 - |-6|}{-1}$
 35. $-7 - \frac{|-14|}{2(2 - 3)} - 3|6 - 8|$ 36. $-7(-3) + \frac{-6}{-3} - |-9|$
 37. $\frac{3|-9 - 2(-3)|}{1 + (-10)}$ 38. $\frac{20(-12) - 40(-15)}{98 - |-98|}$

In Exercises 39–46, determine which of the fundamental laws of algebra is demonstrated.

39. $6(7) = 7(6)$ 40. $6 + 8 = 8 + 6$
 41. $6(3 + 1) = 6(3) + 6(1)$ 42. $4(5 \times \pi) = (4 \times 5)(\pi)$
 43. $3 + (5 + 9) = (3 + 5) + 9$ 44. $8(3 - 2) = 8(3) - 8(2)$
 45. $(\sqrt{5} \times 3) \times 9 = \sqrt{5} \times (3 \times 9)$
 46. $(3 \times 6) \times 7 = 7 \times (3 \times 6)$

In Exercises 47–50, for numbers a and b , determine which of the following expressions equals the given expression.

- (a) $a + b$ (b) $a - b$ (c) $b - a$ (d) $-a - b$
 47. $-a + (-b)$ 48. $b - (-a)$
 49. $-b - (-a)$ 50. $-a - (-b)$

In Exercises 51–66, solve the given problems. Refer to Appendix B for units of measurement and their symbols.

51. Insert the proper sign ($=, >, <$) to make the following true:
 $|5 - (-2)|$ $|-5 - |-2||$
 52. Insert the proper sign ($=, >, <$) to make the following true:
 $|-3 - |-7||$ $||-3| - 7|$
 53. (a) What is the sign of the product of an even number of negative numbers? (b) What is the sign of the product of an odd number of negative numbers?

54. Is subtraction commutative? Explain.

55. Explain why the following definition of the absolute value of a real number x is either correct or incorrect (the symbol \geq means “is equal to or greater than”): If $x \geq 0$, then $|x| = x$; if $x < 0$, then $|x| = -x$.

56. Explain what is the error if the expression $24 - 6 \div 2 \cdot 3$ is evaluated as 27. What is the correct value?

57. Describe the values of x and y for which (a) $-xy = 1$ and (b) $\frac{x - y}{x - y} = 1$.

58. Describe the values of x and y for which (a) $|x + y| = |x| + |y|$ and (b) $|x - y| = |x| + |y|$.

59. The changes in the price of a stock (in dollars) for a given week were $-0.68, +0.42, +0.06, -0.11,$ and $+0.02$. What was the total change in the stock’s price that week?

60. Using subtraction of signed numbers, find the difference in the altitude of the bottom of the Dead Sea, 1396 m below sea level, and the bottom of Death Valley, 86 m below sea level.

61. Some solar energy systems are used to supplement the utility company power supplied to a home such that the meter runs backward if the solar energy being generated is greater than the energy being used. With such a system, if the solar power averages 1.5 kW for a 3.0-h period and only 2.1 kW · h is used during this period, what will be the change in the meter reading for this period? *Hint:* Solar power generated makes the meter run in the negative direction while power used makes it run in the positive direction.

62. A baseball player’s batting average (total number of hits divided by total number of at-bats) is expressed in decimal form from 0.000 (no hits for all at-bats) to 1.000 (one hit for each at-bat). A player’s batting average is often shown as 0.000 before the first at-bat of the season. Is this a correct batting average? Explain.

63. The daily high temperatures (in °C) for Edmonton, Alberta, in the first week in March were recorded as $-7, -3, 2, 3, 1, -4,$ and -6 . What was the average daily temperature for the week? (Divide the algebraic sum of readings by the number of readings.)

64. A flare is shot up from the top of a tower. Distances above the flare gun are positive and those below it are negative. After 5 s the vertical distance (in ft) of the flare from the flare gun is found by evaluating $(70)(5) + (-16)(25)$. Find this distance.

65. Find the sum of the voltages of the batteries shown in Fig. 1.6. Note the directions in which they are connected.

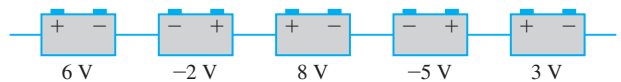


Fig. 1.6

66. A faulty gauge on a fire engine pump caused the apparent pressure in the hose to change every few seconds. The pressures (in lb/in.²) above and below the set pressure were recorded as: $+7, -2, -9, -6$. What was the change between (a) the first two readings, (b) between the middle two readings, and (c) the last two readings?

67. One oil-well drilling rig drills 100 m deep the first day and 200 m deeper the second day. A second rig drills 200 m deep the first day and 100 m deeper the second day. In showing that the total depth drilled by each rig was the same, state what fundamental law of algebra is illustrated.

68. A water tank leaks 12 gal each hour for 7 h, and a second tank leaks 7 gal each hour for 12 h. In showing that the total amount leaked is the same for the two tanks, what fundamental law of algebra is illustrated?

69. On each of the 7 days of the week, a person spends 25 min on Facebook and 15 min on Twitter. Set up the expression for the total time spent on these two sites that week. What fundamental law of algebra is illustrated?
70. A jet travels 600 mi/h relative to the air. The wind is blowing at 50 mi/h. If the jet travels with the wind for 3 h, set up the

expression for the distance traveled. What fundamental law of algebra is illustrated?

Answers to Practice Exercises

1. 3 2. 40 3. 9 4. 2 5. -4 6. 8

1.3 Calculators and Approximate Numbers

Graphing Calculators • Approximate Numbers • Significant Digits • Accuracy and Precision • Rounding Off • Operations with Approximate Numbers • Estimating Results

■ The calculator screens shown with text material are for a TI-84 Plus. They are intended only as an illustration of a calculator screen for the particular operation. Screens for other models may differ.



Fig. 1.7

■ When less than half of a calculator screen is needed, a partial screen will be shown.

■ Some calculator keys on different models are labeled differently. For example, on some models, the EXE key is equivalent to the ENTER key.

■ Calculator keystrokes for various operations can be found by using the URLs given in this text. A list of all the calculator instructions is at goo.gl/eAUGW3.

You will be doing many of your calculations on a calculator, and a *graphing calculator* can be used for these calculations and many other operations. In this text, we will restrict our coverage of calculator use to graphing calculators because a *scientific calculator* cannot perform many of the required operations we will cover.

A brief discussion of the graphing calculator appears in Appendix C, and sample calculator screens appear throughout the book. Since there are many models of graphing calculators, *the notation and screen appearance for many operations will differ from one model to another*. You should *practice using your calculator and review its manual to be sure how it is used*. Following is an example of a basic calculation done on a graphing calculator.

EXAMPLE 1 Calculating on a graphing calculator

In order to calculate the value of $38.3 - 12.9(-3.58)$, the numbers are entered as follows. The calculator will perform the multiplication first, following the order of operations shown in Section 1.2. The sign of -3.58 is entered using the $(-)$ key, before 3.58 is entered. The display on the calculator screen is shown in Fig. 1.7.

38.3 $(-)$ 12.9 \times $(-)$ 3.58 **ENTER** *keystrokes*

This means that $38.3 - 12.9(-3.58) = 84.482$.

Note in the display that the negative sign of -3.58 is smaller and a little higher to distinguish it from the minus sign for subtraction. Also note the * shown for multiplication; the asterisk is the standard computer symbol for multiplication. ■

Looking back into Section 1.2, we see that *the minus sign is used in two different ways*: (1) to indicate subtraction and (2) to designate a negative number. This is clearly shown on a graphing calculator because there is a key for each purpose. The $(-)$ key is used for subtraction, and the $(-)$ key is used before a number to make it negative.

We will first use a graphing calculator for the purpose of graphing in Section 3.5. Before then, we will show some calculational uses of a graphing calculator.

APPROXIMATE NUMBERS AND SIGNIFICANT DIGITS

Most numbers in technical and scientific work are **approximate numbers**, having been determined by some *measurement*. Certain other numbers are **exact numbers**, having been determined by a *definition* or *counting process*.

EXAMPLE 2 Approximate numbers and exact numbers

One person measures the distance between two cities on a map as 36 cm, and another person measures it as 35.7 cm. However, the distance cannot be measured *exactly*.

If a computer prints out the number of names on a list of 97, this 97 is exact. We know it is not 96 or 98. Since 97 was found from precise counting, it is exact.

By definition, $60 \text{ s} = 1 \text{ min}$, and the 60 and the 1 are exact. ■

An approximate number may have to include some zeros to properly locate the decimal point. *Except for these zeros, all other digits are called **significant digits**.*

EXAMPLE 3 Significant digits

All numbers in this example are assumed to be approximate.

- (a) 34.7 has three significant digits.
- (b) 0.039 has two significant digits. The zeros properly locate the decimal point.
- (c) 706.1 has four significant digits. The zero is not used for the location of the decimal point. It shows the number of tens in 706.1.
- (d) 5.90 has three significant digits. *The zero is not necessary as a placeholder and should not be written unless it is significant.*
- (e) 1400 has two significant digits, unless information is known about the number that makes either or both zeros significant. Without such information, we assume that the zeros are placeholders for proper location of the decimal point.
- (f) Other approximate numbers with the number of significant digits are 0.0005 (one), 960,000 (two), 0.0709 (three), 1.070 (four), and 700.00 (five). ■

■ To show that zeros at the end of a whole number are significant, a notation that can be used is to place a bar over the last significant zero. Using this notation, 78,0 $\bar{0}0$ is shown to have four significant digits.

Practice Exercises

Determine the number of significant digits.

1. 1010 2. 0.1010

From Example 3, we see that *all nonzero digits are significant. Also, zeros not used as placeholders (for location of the decimal point) are significant.*

In calculations with approximate numbers, the number of significant digits and the position of the decimal point are important. *The **accuracy** of a number refers to the number of significant digits it has, whereas the **precision** of a number refers to the decimal position of the last significant digit.*

EXAMPLE 4 Accuracy and precision

One technician measured the thickness of a metal sheet as 3.1 cm and another technician measured it as 3.12 cm. Here, 3.12 is more precise since its last digit represents hundredths and 3.1 is expressed only to tenths. Also, 3.12 is more accurate since it has three significant digits and 3.1 has only two.

A concrete driveway is 230 ft long and 0.4 ft thick. Here, 230 is more accurate (two significant digits) and 0.4 is more precise (expressed to tenths). ■

The last significant digit of an approximate number is not exact. It has usually been determined by estimating or *rounding off*. However, it is not off by more than one-half of a unit in its place value.

EXAMPLE 5 Meaning of the last digit of an approximate number

When we write the measured distance on the map in Example 2 as 35.7 cm, we are saying that the distance is at least 35.65 cm and no more than 35.75 cm. Any value between these, rounded off to tenths, would be 35.7 cm.

In changing the fraction $\frac{2}{3}$ to the approximate decimal value 0.667, we are saying that the value is between 0.6665 and 0.6675. ■

■ On graphing calculators, it is possible to set the number of decimal places (to the right of the decimal point) to which results will be rounded off.

*To **round off** a number to a specified number of significant digits, discard all digits to the right of the last significant digit (replace them with zeros if needed to properly place the decimal point). If the first digit discarded is 5 or more, increase the last significant digit by 1 (round up). If the first digit discarded is less than 5, do not change the last significant digit (round down).*